St. Mary's Catholic Primary School
'Dat Deus incrementum'

Dat Deus incrementum
With God we grow.

At St. Mary's we come to flourish and achieve great things through our values of community, pride and joy.

Abstract

This ambitious document captures the scope of our mathematics curriculum that we provide at St. Mary's. Beginning with a vision that focuses on our pupils' futures - that all our stakeholders value. This leads into the rationale behind St. Mary's Mathematics Framework and its six components that establishes a balanced and cohesive approach allowing us to develop pupils who are well rounded mathematicians who can flourish, excel and grow.

Reviewed in October 2022

Next Review January 2023

St. Mary's Mathematics Vision

At St. Mary's we passionately believe in the mathematical potential of all our young people. Learning mathematics at St. Mary's has its own intrinsic value, and our mathematics framework supports this, ensuring that all students will develop an enjoyment of mathematics and an appreciation of its relevance and importance today. In addition, we believe that education should provide pupils with the attitudes, mind-sets and adaptive cognition to allow them to be successful lifelong learners.

Through our vibrant and engaging discussions, our St. Mary's mathematical framework along with our inclusive community will allow all pupils to become confident in their conceptual understanding of mathematics, will reason mathematically and will apply their mathematics to solve challenging problems.

We strive for every pupil, by the end of St. Mary's, to have grown their fluency in the fundamentals of mathematics, their mathematical attitudes and adaptive thinking to achieve their future aspirations enabling them to flourish, have choices and excel. Our high-quality mathematics curriculum enables our pupils to understand the world, that they are part of, allowing them to decide how they want to contribute to our future.

St. Mary's Mathematics Framework

The St. Mary's Mathematics Framework has at its centre our pupils. The framework sets the direction and provides guidance in the teaching, learning and assessment of mathematics from Reception to Year 6. All six components work largely concurrently, not just in sequence. They reinforce one another and the learning on all of them is ongoing.

The framework combines six main components of being a mathematician it allows the for the integration of various research findings from different perspectives into a single framework. This allows us to explore the connections between different types and domains of knowledge and concepts of being a mathematician and acquire a comprehensive view of being a mathematician. Resulting in an ambitious curriculum which offers pupils the opportunities to explore, think and express themselves as mathematicians.

Stakeholders at St. Mary's want to develop all six components of our pupil's mathematical proficiency equally. To achieve this, we help pupils to master mathematics. Mastery in mathematics involves pupils of all ages acquiring a deep, long-term and adaptable understanding of the subject. The phrase 'teaching for mastery' describes the elements of classroom practice and school organisation that combine to give pupils the best chances of mastering mathematics (or any subject). Achieving mastery means acquiring a solid enough understanding of the mathematical concepts and processes that have been taught to enable pupils allow them to move to the next stage in their learning.

At the core of our teaching is the concrete, pictorial and abstract approach.
Concrete – pupils have the opportunity to use concrete objects to help them understand and explain what they are doing.
Pictorial – pupils then build on this concrete approach by using pictorial representations, which can then be used to reason and solve problems.
Abstract – With the foundations firmly laid, pupils can move to an abstract approach using numbers and key concepts with confidence.

We use NCTEM materials, ISee Reasoning, White Rose Maths and information and CPD from our local Maths Hub to support our teaching with a focus on the 'five big ideas' drawn from recent research.

Coherence
Lessons are broken down into small connected steps that gradually unfold the concept, providing access for all pupils and leading to a generalisation of the concept and the ability to apply the concept to a range of contexts.

Representation and Structure
Representations used in lessons expose the mathematical structure being taught, the aim being that pupils can do the mathematics without recourse to the representation as they move forward in their mathematical thinking.

Mathematical Thinking
If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the pupil: thought about, reasoned with and discussed with others.

Fluency

Quick and efficient recall of facts and procedures and the flexibility to move between different contexts and representations of mathematics.

Variation

Variation is twofold. It is firstly about how the teacher represents the concept being taught, often in more than one way, to draw attention to critical aspects, and to develop deep and holistic understanding. It is also about the sequencing of the sessions, activities and exercises used within a lesson and follow up practice, paying attention to what is kept the same and what changes, to connect the mathematics and draw attention to mathematical relationships and structure.

The Five Big Ideas were first published by the NCETM in 2017.

Successful mathematicians understand the connections across the curriculum, are fluent in mathematical facts and skills, can utilise heuristics and procedures to reason and solve problems while explaining their thinking. All the while having a positive attitude about themselves as learners and enjoying mathematics. Looking for and reflecting on the mathematics in their environment.

Our Curriculum Progression @ St. Mary's

Progression Map

We have a Progression Map that has been developed through research and supported along the way from external providers. This Progression Map incorporates a carefully planned progression of knowledge to ensure depth and breadth of learning in mathematics.

Resulting in every pupil having the opportunity to achieve the six 'end points'. These 'end points' correspond to the six components of our St. Mary's Mathematics Framework. For each 'end point' the specific knowledge required has been systematically planned for each year group to allow for an epistemic ascent of knowledge. The concepts already understood by pupils are brought into new relations of abstraction and generality, giving them yet more power to challenge, rethink and create.

End Point One - Attitudes & Mindset

Pupils have developed a mathematical mindset and established a positive attitude towards mathematics. That allows them to appreciate the importance, beauty and awe of mathematics in the world and its usefulness in modern life; have had the opportunity to utilise their mathematical thinking and reasoning skills.

End Point Two - Conceptual Understanding

Pupils have acquired a conceptual understanding of mathematics through a process of epistemic ascent, where they are proficient in solving problems; have a deep understanding of the structures, visual patterns and calculation strategies required to succeed with the KS3 curriculum and beyond.

End Point Three - Communication & Language

Pupils are able to communicate, explain, question, justify and prove their understanding using mathematical terminology and notation. They have the fluency in the vocabulary required to describe and illustrate mathematical processes, concepts and their thinking. Mathematics is a language of seeing the world with new eyes.

End Point Four - Heuristics, Problem Solving & Reasoning

Pupils can explore novel mathematical situations and construct solutions using a bank of strategies and the confidence to select the most appropriate and efficient one for any given routine or non-routine task across different contexts.

End Point Five - Fluency

Pupils are able to fluently recall numerical calculations, measurement facts, algebraic manipulations, spatial visualisation, estimation and many other mathematical facts required to succeed with the KS3 curriculum and beyond.

End Point Six – Adaptive Cognition

Pupils are motivated to use their adaptive cognition and meta-cognition to plan, monitor and evaluate their deep mathematical thinking. Able to make strategic choices, links, generalisations and build connections across big mathematical ideas, domians, subjects and the world.

St. Mary's Mathematics Framework – *End Points*

Attitudes & Mindsets
Pupils have developed a mathematical mindset and established a positive attitude towards mathematics. That allows them to appreciate the importance, beauty and awe of mathematics in the world and its usefulness in modern life; have had the opportunity to utilise their mathematical thinking and reasoning skills.

Heuristics, Problem Solving & Reasoning
Pupils can explore novel mathematical situations and construct solutions using a bank of strategies and the confidence to select the most appropriate and efficient one for any given routine or non-routine task across different contexts.

Conceptual Understanding
Pupils have acquired a conceptual understanding of mathematics through a process of epistemic ascent, where they are proficient in solving problems; have a deep understanding of the structures, visual patterns and calculation strategies required to succeed with the KS3 curriculum and beyond.

At St. Mary's we grow flourishing mathematicians

Adaptive Cognition
Pupils are motivated to use their adaptive cognition and meta-cognition to plan, monitor and evaluate their deep mathematical thinking. Able to make strategic choices, links, generalisations and build connections across big mathematical ideas, strands, subjects and the world.

Fluency
Pupils are able to fluently recall numerical calculations, measurement facts, algebraic manipulations, spatial visualisation, estimation and many other mathematical facts required to succeed with the KS3 curriculum and beyond.

Communication & Language
Pupils are able to communicate, explain, question, justify and prove their understanding using mathematical terminology and notation. They have the fluency in the vocabulary required to describe and illustrate mathematical processes, concepts and their thinking. Pupils appreciate mathematics is a language to see the world with new eyes.

Sources: Drury H. (2014) Mastering Mathematics, Teaching to transform achievement, OUP ISBN: 978-0-19-835175-7; IOE, Singapore (1990-2003); McCourt M. (2019) Teaching for Mastery; Sommerhoff D. (2017); NCETM Maths Hub

St. Mary's Mathematics Framework

Attitudes & Mindsets

Heuristics, Problem Solving & Reasoning

Conceptual Understanding

At St. Mary's we grow flourishing mathematicians

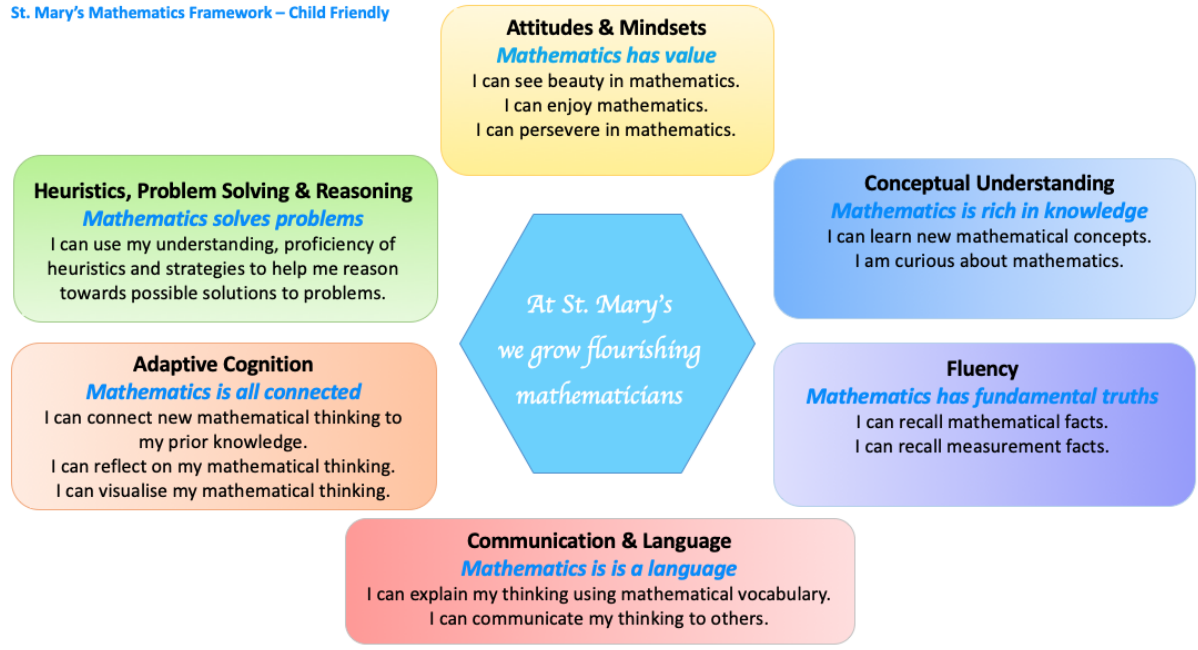
Adaptive Cognition

Fluency

Communication & Language

Sources: Drury H. (2014) Mastering Mathematics, Teaching to transform achievement, OUP ISBN: 978-0-19-835175-7; IOE, Singapore (1990-2003); McCourt M. (2019) Teaching for Mastery; Sommerhoff D. (2017); NCETM Maths Hub

St. Mary's Mathematics Framework – Child Friendly



Mathematics has value
I can see beauty in mathematics.
I can enjoy mathematics.
I can persevere in mathematics.

Mathematics is is a language
I can explain my thinking using mathematical vocabulary.
I can communicate my thinking to others.

Mathematics solves problems
I can use my understanding to solve problems.
I can use my understanding, proficiency of heuristics and strategies to help me reason towards possible solutions to problems.

Mathematics is rich in knowledge
I can learn new mathematical concepts.
I am curious about mathematics.

Mathematics is all connected
I can connect new mathematical thinking to my prior knowledge.
I can reflect on my mathematical thinking.
I can visualise my mathematical thinking.

Mathematics has fundamental truths
I can recall mathematical facts.
I can recall measurement facts.

Attitudes & Mindsets

Attitudes refer to the affective aspects of mathematical learning. Mindsets can change your attitude. Mindsets are a set of key beliefs you hold. These beliefs are a collection of thoughts that shape your thought habits. In turn these thought habits affect how you think, what you feel and what you do. Attitude is when these ways of thinking or feeling about something are settled. One's attitude is reflected in a person's behaviour (Jaiswai, N. 2017).

Aside from physical and health needs and school readiness, pupils' ability to learn are determined by their Mindsets. Research has identified four Mindsets required for learning, these deeply influence pupils' behaviour, outcomes and overall drive to learn (Farrington, et al., 2012):

- Self-Efficacy
- Sense of Belonging
- Relevance
- Growth Mindset
- Logical & methodical

"I can succeed."

Self-Efficacy is the belief that one can succeed (Bandura, A. 1986). Pupils must believe that they can achieve their goals, however they define those goals. If pupils think they need help or resources, they must see a path they can take in order to obtain the required help or resource to complete a particular task (Brigman, G. & Webb, L. 2007; Oyserman, D., Bybee, D., & Terry, K. 2006).

"I belong in this learning community."

Sense of Belonging is when pupils feel they belong to a learning community, they become engaged in learning (Harvey & Schroder, 1963; Oyserman, D., Bybee, D., & Terry, K. 2006). This involves seeing peers as people who value going beyond one's comfort zone (Vygotsky, 1978) and learning about mathematics (about anything), as well as feeling connected to those peers. Learning is seen as "cool" - when this is evident, pupils associate learning with social rewards they value (Walton & Cohen, 2007; Cohen et al. 2006; 2009).

"This work has value and purpose for me."

The **relevance** of learning and subsequently the relevance of school impacts on the engagement of pupils. Pupils engage in learning much more energetically and deeply when they value the knowledge and skills that they're working to acquire, or find them beautiful, relevant or interesting (Eccles et al., 1983). That leads them to think deeply, question, pursue, and put their full selves into their work. Their appreciation of the beauty and power of mathematics can result in an enjoyment in learning mathematics. Project-based learning and real-world connections are ways to foster relevance and help pupils explore passions, goals, and applications of learning. We can also have pupils reflect on and write about the relevance of their work or a learning experience they're about to embark on (Hulleman & Harackiewcz 2009).

"I can change my intelligence and abilities through effort."

Pupils with a **growth mindset** understand that their intelligence and abilities to think can change through supportive effort (Mueller, C. & Dweck, C. 1998). It is having a belief about the nature of human abilities, that they are malleable/changeable. That we are works in progress and can continue to develop ourselves at anything and at any age (Briceño, E. 2012). Like with all Mindsets they can be learnt, when modelled. Pupils work harder because they want to do so, they use more effective strategies, and they reach higher levels of achievement (Aronson, Fried, & Good 2002; Blackwell L., et. al., 2007). While this benefits all pupils, it also breaks down negative stereotypes and thereby contributes to closing the achievement gap (Good C., Aronson J., & Inzlicht M., 2003).

We may think that pupils implicitly learn these skills as they do their work, but without explicit instruction and reflection, this understanding is not as solidly consolidated in the mind and understood (Schwartz, 1998). Research shows that we need experiences to deeply understand knowledge, but we also need explicit discussion and reflection to truly make sense of our experiences and apply our new understanding in different contexts.

Implementing these five Mindsets

Staff need to model these five Mindsets
Pupils need to emulate and embed …
Pupils need to articulate …
Pupils need to write about …
Pupils need to practice …
Pupils need to reflect on …

Pupils' attitudes towards mathematics are shaped by their learning experiences. At St. Mary's we provide them with opportunities where the learning of mathematics is fun, meaningful and relevant which helps them develop those positive attitudes towards mathematics. Care and attention is given to the design of learning sequences to build their confidence and develop an appreciation for mathematics. We want pupils to gain enjoyment as result of their flourishing self-confidence in their growing abilities.

Above all, pupils' beliefs can influence their attitudes to learning, especially in independent learning where pupils are encouraged to take on more responsibility for their learning. Mathematics has value and it makes sense. At St. Mary's we want pupils to enjoy mathematics and become better at it by persevering.

Next steps:

- Re-visit for the context of Mindsets and Staff learning **Source**: YOUCUBED MINDSESTs
- Logical & methodical Mindsets **Source**: OFTSED May 2021 Review

Adaptive Cognition

Cognition is a broad term that at its simplest refers to thinking. It is the psychological processes involved in the acquisition and understanding of knowledge.

Metacognition is the act of thinking about thinking, the awareness of and the ability to control one's thinking processes. In particular when making connections to prior learning and deciding which heuristic or strategy to use when solving a problem. This includes planning and monitoring of one's own thinking.

To develop metacognitive awareness and strategies, and know when and how to use the strategies, students should have opportunities to solve non-routine and open-ended problems, to discuss their solutions, to think aloud and reflect on what they are doing, and to keep track of how things are going and make changes when necessary. This is modelled by the staff who verbalise their thinking processes when tackling a problem.

Adaptive cognition is the ability to transition between these two states. To illustrate this, I will use C. S. Lewis's "Meditation in a Toolshed":

> *...at the start he imagines entering a dark toolshed. From a crack above comes a sunbeam. He stands staring at the specks of dust floating in it mesmerised. Everything else was pitch-black. He was seeing the beam, not seeing things by it. This would be our beyond thinking – metacognition. Then he moves into the light so it falls on his face. He sees no toolshed, and above all no beam. Instantly the whole picture has changed. Through the crack above he sees green leaves moving on the branches of a tree outside and beyond that 90 odd million miles away, the sun. Looking along the beam, and looking at the beam are very different experiences.*

If we imagine the beam of light is cognition, looking at our cognition from a distance would be equivalent to metacognition. Stepping into that beam and looking deeply into it would be deep cognition – deep thinking – when pupils are learning or overlearning. At St. Mary's we develop pupils to have adaptive cognition, moving between these two states.

Deep cognitive mathematics is the process of thinking deeply about mathematics. This ability allows pupils to see connections and make links among mathematical ideas, between components, domains and other subjects, and the connections between mathematics and the world. This helps pupils make schema that are both broad and deep allowing them to make sense of what they will learn in mathematics in the future.

Next steps:

- Cognitive activators

Communication & Language

Mathematics is a universal language, that helps us understand our world. Communication refers to the ability to use mathematical language to express mathematical ideas and arguments concisely, logically and precisely. Pupils deepen their understanding by explaining, justifying and asking questions while using mathematical language. This use of language also acts as a scaffold for their learning with their peers, parents and teachers (Vygotsky, 1978). Pupils are expected to answer in full sentences as modelled by the staff.

Pupils vocabulary is progressive and uses their schema and linguistic structures thereby easing working memory (and its subsystems: phonological loop, visuo-spatial sketch pad (VSSP) and central executive).

Working memory has been found to be crucial when students are integrating knowledge from the linguistics and quantitative pathways. However, as pupils progress through St. Mary's they increasingly use verbal representations of quantities such as number words and the role of VSSP has less impact, therefore less impact on working memory.

Next steps:

- Dialogic Education, 4Cs, Oracy
- Vocabulary word mats and class dictionaries
- Cognitive content dictionaries to develop

Conceptual Understanding

Mathematical concepts can be grouped into:

- number and place value,
- calculations,
- fractions, decimals and percentages
- ratio and proportion
- algebra
- measurement
- geometry
- statistics

However, these strands and their sub-strands are connected and interdependent.

At different stages of learning the breadth and depth of the content varies. For pupils to develop a deep understanding of mathematical concepts, and to make sense of mathematical ideas pupils will be exposed to a variety of learning experiences that use the CPA Approach to help them relate abstract mathematical concepts with concrete and pictorial experiences. This requires explicit modelling, discussion and reflection to make sense of these experiences allowing pupils to apply this new learning and understanding in novel contexts.

Next steps:

- Dialogic Education

Fluency

Fluent in the fundamental mathematical facts and skills such as numerical calculations, algebraic manipulation, spatial visualisation, data analysis, measurement, use of tools and estimation. Becoming automatic in these skills is important prerequisite for more complex learning and helps with problem solving as it reduces the cognitive load. These skills would also include the abilities to use spreadsheets and other software to utilise mathematics when solving problems.

These skills need to be varied and frequently practiced to develop the ability to recall and apply knowledge rapidly and accurately. These skills should be taught alongside the conceptual understanding of the underlying mathematical principles where possible and not as mere procedures.

To develop pupils' fluency and mental mathematics skills, we are introducing Key Instant Recall Facts (KIRFs) throughout school. KIRFs are a way of helping our pupils to overlearn key facts and information which they need to have instant recall of.

KIRFs are designed to support the development of mental mathematics skills that are the keystones for the subsequent mathematics across our school. They are particularly useful when calculating, adding, subtracting, multiplying or dividing. They contain number facts such as number bonds and times tables that need constant practise and rehearsal, so pupils can recall them quickly and accurately.

Instant recall of facts helps enormously with mental agility in mathematics lessons. When pupils move onto written calculations, knowing these key facts is very beneficial. For our puils to become more efficient in recalling them easily, they need to be practised frequently and for short periods of time. KIRFs are over learnt as they are considered pre-requisites for subsequent year groups.

Each half term, pupils will focus on a KIRF to practise and learn at home for the half term. They will also be available on our school website under the maths section and each pupil will receive a copy to keep at home. We will also have a permanent 'Maths Mastery' display in our corridor. The KIRFs include practical ideas to assist pupils in grasping the key facts and contain helpful suggestions of ways in which parents and family could make this learning interesting and relevant. They are not designed to be a time-consuming task and can be practised anywhere – in the car, walking to school, etc. Regular practice - little and often – helps children to retain these facts and keep their skills sharp. Throughout the half term, the KIRFs will also be practised in school during Fluency and RPL (Review Previous Learning) sessions and staff will assess whether pupils can recall these KIRFs.

Over their time at St. Mary's, we believe that - if the KIRFs are developed fully - pupils will be more confident with number work, understand its relevance, and be able to access the curriculum much more easily. They will be able to apply what they have learnt to a wide range of problems that confront us regularly.

At St. Mary's we value Fluency in mathematics and have chosen to protect a portion of the school day for its explicit teaching and pupil practice. These sessions are a mixture of addressing the KIRFs, overlearning mathematics that has been previously taught in the main mathematics lesson or follows the No Nonsense Number Facts progression.

Next steps:

- Cognitive Load
- Dual Coding
- Precision Teaching of the Key Instant Recall Facts (KIRFs)
- Number Facts Years 1-6
- Number Sense
- NNF

Heuristics & Reasoning

Reasoning mathematically and heuristics are essential for mathematical problem solving. Mathematical reasoning refers to the ability to explore mathematical situations and construct solutions and logical arguments. This can be developed through the application of routine and non-routine problems in different contexts.

Exploring, classifying, comparing, spatial visualisation, working systematically, thinking strategically, questioning, conjecturing, visualising, modelling, representing, justifying, proving are all at the heart of reasoning mathematically.

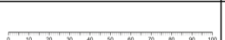
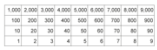
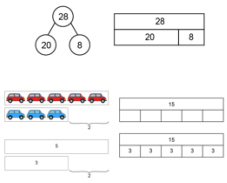
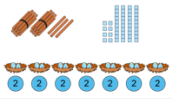
Heuristics are a bank of strategies that can be used to solve a problem. These include:

- representations – e.g. drawing a sketch, graph or table, creating an organised list
- making a guess – e.g. guess and check, trial and error, making a hypothesis
- walking through the problem – e.g. acting out the situation, using concrete materials, working backwards, using logic and cues
- changing the problem – e.g. using simpler numbers to solve the problem, breaking the problem into smaller parts, writing a number sentence, looking for patterns

Some problems can be solved in several ways using different heuristics, while others are solved using a combination of heuristics.

Thinking mathematically is the state required when solving a problem, as opposed to thinking deeply about Mathematics.

At St. Mary's we have core representations that all pupils will have multiple opportunities to learn, explore and use across their time at our school. These are integrated into our progression and are supplemented by the ISee Visual representations.

Representation		Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Tens frame	1NPV-1 1AS-1 1 NF-1	2AS-1 2AS-3	3NPV-1 3NF-1 3NF-3	4NPV-1 4NF-3	5NPV-1 5NF-2 5MD-1	6NPV-1
	Number line	1NPV-1 1NPV-2 1NF-2	2NPV-2 2AS-2	3NPV-3 3F-3 3F-4	4NPV-3 4F-1 4F-2 4F-3	5NPV-3 5F-2 5F-3	6NPV-3 6F-1
	Gattegno chart	1NPV-1 1NF-2			4MD-1	5NPV-2 5MD-1	6NPV-1
	Partitioning diagrams including bar models	1AS-1 1AS-2 1NF-1	2NPV-1 2AS-1 2AS-3 2AS-4	3NPV-2 3NPV4 3AS-1 3AS-2 3AS-3 3F-2 3F-4	4NPV-2 4NPV-4 4MD-2 4F-3	5NPV-2 5NPV-4 5F-1	6NPV-4 6AS/MD-4 6F-3
	Groups of units in addition to ones such as Dienes, PV counters		2NPV-1 2AS-3 2AS-4 2MD-1 2MD-2	3AS-2 3MD-1	4MD-2 4F-2	5 NPV-1 5MD-3 5MD-4	6NPV-2

Impact of the curriculum on our stakeholders

What does it mean for staff?

The six components of the mathematics framework are integral parts of mathematics learning and problem solving. The intent of the framework is to help staff focus on these in their teaching practice so as to provide a more enriching, engaging, student-centred learning environment and to promote greater diversity and creativity in learning.

What does it mean for parents?

Parents are able to engage with their child's learning and support them.

What does it mean for Governors?

Governors are able to question and monitor our ambitious mathematical curriculum.

What does it mean for wider community?

Pupils leaving St. Mary's who are proficient in mathematics and are able to share their learning and enjoyment of it with the world.

What does it mean for our pupils?

That they enjoy and value mathematics.

What has COVID-19 and subsequent lockdowns mean for the curriculum pupils have received?

- *For the 2021 Summer term we chose to focus on the Ready-to-Progress statements as our core teaching. Along with the MTP progression the subsequent small steps were included and shared with teachers. Fluency across KS2 around KS1 KIRFs was acknowledged as an area of need. This was followed by pupil assessment and with class or individual interventions.*

- *For 2021-2022 academic year we chose to use the NCTEM Prioritisation Curriculum that has been adapted for our context and pupils based off the previous teachers guidance and ongoing formative assessments. The Fluency across KS2 has improved significantly but class and individual interventions continue.*

Different types of knowledge

The knowledge within the Progression Map has been codified into groups of knowledge. Substantive and disciplinary which are illuminating categories not only for understanding the curriculum but also for grasping the implications of curriculum for teaching and assessment.

The substantive knowledge refers to the big ideas of mathematics and has three sub-groups of knowledge:

- Declarative knowledge
  - "I know that …" or " … to know that …"
- Procedural knowledge
  - "I know how …" or " … to know how …"
- Conditional knowledge
  - "I know when …" or " … to know when …"

Disciplinary knowledge is the pursuit of mathematical truth, how the knowledge of mathematics has been acquired. It is through due attention to the disciplinary dimension that pupils know that what we teach is not all that there is. As educators we have to show disciplinary attentiveness by modelling responsible claims.

- "I know why …" or "… to know why …"
- "I know who …" or " … to know who …"
- "I know how …" or " … to know how …"
- "I know why/how the knowledge was established/acquired"

These knowledge statements increase in complexity through the key stages. To allow for the way in which our knowledge alters the long-term memory of our pupils over subsequent revisits, in addition to increasing their curiosity and capacity for new knowledge.

Regarding teaching, they help teachers and senior leaders to interpret teaching activities in the light of an object. Before one can apply research into the efficacy of (say) pair/group discussion, one needs to establish what is being taught. Therefore, the assessment of this knowledge will be different.

A world of difference exists between a paired discussion designed to practise a facet of open argument derived from a particular discipline and a paired discussion designed for learning substantive content. In one, the dialogue teaches a disciplinary process; in the other, the rationale is constructivist pedagogy. They cannot be appraised in the same way.

However, having a knowledge-rich curriculum which incorporates the breadth and depth of substantive and disciplinary knowledge across six components does not mean the curriculum is fixed. As responsible educators we are continually asked to be 'curriculum making' not just transmitters of knowledge. We must keep our focus on the pedagogic strategies that will facilitate better access for all pupils to the knowledge within the progression map and not on the examination of the knowledge

The progression map has a third domain of knowledge know as affective knowledge that refers to the pupils' feelings, emotions and attitudes towards mathematics and learning.

Reception

Teachers follow a similar small steps progression as the rest of the school that has been constructed using the Development Matters, the Early Childhood Maths Group (ECHG), NCTEM, and Numberblocks resources. Pupils are introduced to maths concepts through whole class teaching, before moving on to teacher-led and pupil-led tasks within our indoor and outdoor provision.

The first few years of a child's life are especially important for mathematics development. Research shows that early mathematical knowledge predicts later reading ability and general education and social progress (Duncan et al, 2007). Conversely, children who start behind in mathematics tend to stay behind throughout their whole educational journey (Aubrey, Godfrey, Dahl, 2006).

The objective for those working in Early Years, then, is to ensure that all children develop firm mathematical foundations in a way that is engaging, and appropriate for their age. The six concepts are primarily designed to support Reception teachers (those working with 4-5 year olds), and are based on international research.

There are six key areas of early mathematics learning, which collectively provide a platform for everything children will encounter as they progress through their maths learning at primary school, and beyond.

The materials below were first published by the NCETM in 2018 and updated in 2019.

The progression has six concepts that plans for the depth of understanding required by pupils in early years these are:

- Cardinality & Counting
- Comparison
- Composition
- Change
- Spatial Reasoning & Pattern
- Measurement

Cardinality & Counting

Understanding that the cardinal value of a number refers to the quantity, or 'how manyness' of things it represents.

Comparison

Understanding that comparing numbers involves knowing which numbers are worth more or less than each other.

Composition

Understanding that one number can be made up from (composed from) two or more smaller numbers.

Change

Understanding combining, separating groups of objects and increasing as adding to (augmentation) and decreasing as taking from/away (reduction).

Spatial Reasoning

Understanding what happens when shapes move, or combine with other shapes, helps develop wider mathematical thinking.

Pattern

Looking for and finding patterns helps children notice and understand mathematical relationships.

Measurement

Comparing different aspects such as length, weight and volume, as a preliminary to using units to compare later.

Numberblocks

These resources form the fundamental mathematical basis of a CBeebies series of five-minute animated programmes. The NCETM has provided support materials linked to the Numberblocks programmes. These are designed to help Early Years practitioners draw out and build on the maths embedded in the stories contained in each episode. At St. Mary's we have integrated these into our small steps progression document for Early Years.

Domains of Knowledge across the St. Mary's Mathematics Curriculum

Domains of Knowledge within **Conceptual Understanding**:

- Number (including parts and wholes/integers and decimals)
- Calculation (including the four operations) and proportion
- Measurement (including money and time)
- Geometry (including shape, position and direction)
- Statistics
- Algebra

Domains of Knowledge within **Heuristics & Reasoning Problem Solving:**

- Mathematical heuristics and reasoning for problem solving
    - Act it Out
    - Use a diagram/model
    - Make a systematic list
    - Look for pattern(s) and extending it
    - Work backwards
    - Use before-after concept
    - Use guess and check
    - Make suppositions
    - Restate the problem in another way
    - Solve part of the problem
    - Thinking of a related problem
    - Use equations
    - Draw a sketch, graph or table
    - Using concrete materials
    - Simplify the problem - using simpler numbers to solve the problem, then applying the same methodology to the real problem
    - Writing a number sentence

Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments. It is a habit of mind that can be developed through the applications of mathematics in different contexts.

Heuristics are what students can do to approach a problem when the solution to the problem is not obvious. Some examples of heuristics are listed below in four categories according to how they are used:

- To give a representation – e.g. draw a diagram, make a list
- To make a calculated guess – e.g. guess and check, look for pattern(s), make suppositions
- To go through the process – e.g. act it out, work backwards, before-after
- To change the problem – e.g. restate the problem, simplify the problem, solve part of the problem

Domains of Knowledge within **Adaptive Cognition:**

- Mathematical thinking and meta-cognition
    - Classifying
    - Comparing
    - Sequencing
    - Analysing parts and whole
    - Identifying patterns and relationships
    - Induction
    - Deduction
    - Generalising
    - Verifying
    - Spatial Visualisation

Thinking skills can be used in the thinking process such as classifying, comparing, sequencing.

- Meta-cognition
    - awareness of and the ability to control one's thinking processes
        - monitoring of one's own thinking
        - self-regulation of learning

Meta-cognition embraces two aspects. the monitoring aspect – monitoring of one's own thinking – requires pupils to know the meta-cognitive strategies, and when and how to use them.

- Expose pupils to general problem-solving skills, thinking skills, and heuristics, and how these skills can be applied to solve problems.
- Encourage pupils to think aloud the strategies and methods they use to solve particular problems.
- Provide pupils with problems that require planning (before solving) and evaluation (after solving)
- Encourage pupils to seek alternative ways of solving the same problem and to check the appropriateness and reasonableness of answers
- Allow pupils to discuss how to solve a particular problem and to explain the different methods that they can use for solving problems

2nd Order Concepts

- Fluency
    - Procedural skills for numerical calculations
    - Key Instant Recall Facts (KIRFs)
    - Measurement
    - Times Tables
    - Estimation
    - Algebraic manipulations
    - Spatial visualisations
- Attitudes and Mindsets
    - affective aspects of mathematics learning:
        - beliefs about mathematics and its usefulness
        - interest and enjoyment in learning mathematics
        - appreciation of the beauty and power of mathematics
        - confidence in using mathematics
        - perseverance in solving a problem
        - Positive Mindset when making mistakes and learning from them

Pupils' attitudes towards mathematics are shaped by their learning experiences. making the learning of mathematics fun, meaningful and relevant goes a long way to inculcating positive attitudes towards the subject. Care and attention should be given to the design of learning activities to build confidence in and develop appreciation for the subject. Perseverance when solving non-routine problems and open-ended problems which require pupils to investigate and solve using a wide range of heuristics. Pupils with this important quality will not give up easily when they encounter difficulties in solving a problem. Pupils' beliefs about mathematics and its usefulness can influence their attitudes in mathematics learning and problem solving. This dimension is desirable for pupil-centred learning where pupils are encouraged to take on more responsibility for their learning.

- Communication & Language
    - Mathematical vocabulary
    - Mathematical notation
    - Listening
    - Oracy
    - Explaining and justifying
    - Written, artwork and performing

Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps to scaffold pupils' thinking, develop their own understanding of mathematics and sharpen their mathematical thinking.

Key Concepts within these Domains of Knowledge

Number (including parts and wholes/integers and decimals)

- Wholes and parts/ Integers and fractions; equivalent fractions, decimals and percentages (FDP), calculating with FDP, improper and mixed number fractions, unit and non-unit fractions.
- Value and Equivalence; sameness and difference, comparison and ordering, expanded notation, conversion.

Calculation (including the four operations) and proportion

- Additive thinking; addition and subtraction, composition and decomposition, mental methods, written methods, equal difference.
- Multiplicative thinking; times tables, multiplication and division, unitising, doubling and halving, commutative law, associative law, distributive law, mental methods, written methods, prime numbers, squared and cubed numbers, scaling, ratio.
- Mental methods of calculation; number bonds, estimation, substitution, rounding and adjusting, bridging, composition and decomposition.

Measurement (including money and time)

- Measuring time, distance, turns, weight, capacity, currency with accuracy.
- Digital and analogue time, 12 and 24hr time.
- Equivalency and conversion of measurements.

Geometry (including shape, position and direction)

- Names and properties of 2D and 3D shapes, classification and sorting.
- Types of lines.
- Transformation of shapes.
- Measuring, drawing and reasoning with angles.
- Perimeter, area and volume including composite and rectilinear shapes, using appropriate units.

Statistics

- Read and interpret data in bar charts, pictograms, line graphs, timetables, time graphs, conversion graphs and tables.
- Present data appropriately.

Algebra

- Knowns and unknowns.
- Substitution, representation, deriving facts.
- Order of operations.

Mathematical heuristics and reasoning for problem solving

- Working methodically
- Representation
- Substitution
- Pattern Identification
- Organisation
- Modelling
- Logic, Equality (=)
- Presentation

Mathematical thinking and meta-cognition

- Deduction
- Efficiency
- Collaborative cognition
- Meta-cognitive awareness
- Creative, Caring, Critical
- Self-regulation of learning
- Shared Cognition

Pedagogy @ St. Mary's

Mastery of Mathematics @ St. Mary's

St. Mary's Mathematical Framework will be implemented using the mastery model (Bloom, B. 1968; Guskey, 2010). A mastery model has the following core elements.

- Diagnostic pre-assessment with pre-teaching
- High-quality, group-based initial instruction with class discussion
- Progress monitoring through regular formative assessment
- High-quality corrective instruction
- Second, parallel formative assessments
- Enrichment or extension activities

The Components of Mastery Mathematics

- Concrete, Pictorial, Abstract (CPA) Approach
- Contextualise the mathematics
- Number Bonds
- Bar Modelling
- Core Representations
- Developing fluency
- Activating/orientating prior Learning
- New Learning in small steps using the NCETM Primary Mastery Professional Development Materials and the Ready to Progress Guidance
- Consolidation of Learning
- Retrieval
  Purposeful intelligent practice
- Variation
  - Intelligent practice
  - Use empty box problems
- Internal Mathematics
  We use the term internal mathematics for the mathematical thinking that we can engage in without physical aids. This is also referred to as mental mathematics.

Impact @ St. Mary's

St. Mary's Mathematics Curriculum

To develop at least 80% of St Mary's pupils' proficiency so they can grow and flourish as mathematicians by 2024.

To be securing the relevant KIRFS for the pupils who are working towards their year group expectations by end 2023.

Attitudes & Mindsets

Conceptual Understanding

Fluency

Heuristics, Problem Solving & Reasoning

Adaptive Cognition

Communication & Language

R2P Guidance integrated into MTP

St. Mary's Lesson

Lesson Pro-forma

Fluency

Core Models

Cloze & Stem Sentences

NCETM SPINES PD

bonds Catch Up

Retrieval Practice

Strand Focus

KIRFS Precision Teaching

Growth Mindset Lessons 2022

Ofsted Review May '21

Sumdog engagement

Keep Up

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